

Polarization and Campaign Spending in Elections¹

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Abstract

We develop a Downsian model of electoral competition in which candidates with both policy and office-motivations use a mixture of platforms and campaign spending to gain the median voter's support. The unique equilibrium involves randomizing over both platforms and spending, and exhibits the following properties – (i) ex-ante uncertainty in platforms, spending, and the election winner, (ii) platform divergence, (iii) inefficiency in spending and outcomes, (iv) polarization, and (v) voter extremism. We also show that platform polarization and campaign spending move in tandem, since spending is used by candidates to gain support for extreme platforms. Factors that contribute to both phenomena include the candidates' *desire* for extreme platforms, and their *ability* to translate campaign spending into support for them. The latter insight generates new hypotheses about the potential causes of both rising polarization and spending.

Two of the most striking features of American elections are the growing polarization of candidates (Hall (2018)) and the ever-increasing amounts of campaign spending (Meirowitz (2008)), leading several scholars to speculate that these phenomena are linked (McCarty, Poole and Rosenthal (2006)). We develop a model that organizes a collection of empirical regularities, offers a plausible rationale for this link, and generates new hypotheses about the potential causes of both phenomena.

We build on the classic Downsian model of electoral competition, in which two candidates simultaneously commit to policy platforms. In equilibrium the candidates converge to the median voter’s ideal policy, even if they are partially or wholly ideologically motivated. We add the ability to make costly campaign expenditures; in our model each candidate simultaneously chooses how much to spend alongside her choice of platform. Spending improves the voters’ evaluation of her non-ideological attributes or *valence* (e.g. Ashworth and Bueno de Mesquita (2009); Groseclose (2001); Meirowitz (2008); Serra (2010); Wiseman (2006)), as in previous works studying campaign spending.

The main strategic force in our model is that ideologically-motivated candidates (Calvert (1985)) exploit spending to gain support for more extreme platforms. We show that this produces five striking equilibrium effects. The first is *electoral uncertainty* – both the platforms of candidates, and the behavior of voters, are ex-ante unpredictable. The second is *platform divergence* – both candidates position strictly away from the median. The third is *inefficiency* – candidates always waste money trying to win, and there is uncertainty over the final policy outcome that harms voters and candidates. The fourth is *polarization* – candidates are most likely to take extreme positions far from the median, and outcomes are most likely to be far as well. The fifth is *voter extremism* – the median paradoxically selects the candidate with the most extreme platform due to this candidate’s excessively high spending.

We next examine how the candidates’ platform polarization and spending are affected by the candidates’ motivations, the voters’ motivations, and the cost of raising funds and influencing voters with them. Across all comparative statics we find that polarization and spending move together, thereby offering a plausible explanation for the connection between them; polarization reflects the candidates’ willingness to exploit spending to gain support for more ideologically extreme platforms.

Two factors simultaneously contribute to both phenomena. The first is the candidates’ *desire* to influence policy outcomes, determined by both their ideological extremism and ideological intensity. We thus reproduce an existing hypothesis about the cause of rising polarization – that candidates are

taking more ideologically-extreme platforms because they have intrinsically more extreme preferences (Fiorina (1999)). (Note that more extreme preferences do not generate more extreme platforms in the classic Downsian model). The second is the candidates’ *ability* to translate spending into support for extreme platforms. This ability is in turn the product of three factors. The first is the marginal cost of a dollar of campaign spending, which is naturally interpreted as the marginal cost of fundraising. This is influenced by the candidates’ (dis)taste for fundraising, their skill (or lack thereof) at doing so, and their ability to target potential donors. The second is the marginal impact of a dollar of spending on voters; this is influenced by the dollar cost of traditional campaign activities like advertising and canvassing, their effectiveness in conjunction with new tools like microtargeting, and the availability of new persuasive strategies like social media. The third is the strength of the votes’ ideological commitments, which influences how much spending is needed to “compensate” them for ideologically extreme platforms. If spending becomes cheaper or more effective through any of these channels (or a combination), our model predicts that an increase in both polarization and spending will result.

We last examine welfare. Results about voter welfare hinge crucially on whether the valence generated by campaign spending actually improves welfare – because it reflects “character valence” attributes like competence and expertise (Stone and Simas (2010)) – or simply proxies for how spending biases voters. Assuming the latter, spending competition harms voters, and factors that increase polarization reduce welfare; this is in line with the conventional wisdom about polarization. Assuming the former, spending competition actually *helps* voters, and factors that increase polarization *increase* voter welfare, due to the higher candidate valence that also results. A surprising implication is that constraints on fundraising and spending would actually harm voters despite causing platforms to moderate. The candidates, in contrast, are unambiguously harmed by campaign spending, and become worse off as their desire or ability to influence policy outcomes goes up. For them, “dialing for dollars” is a wasteful race to the bottom that both would prefer to avoid, but neither can commit to.

Related Literature Our model joins a literature studying the choice of platforms and spending. Our most notable difference is the sequence – candidates choose platforms and spending simultaneously, rather than platforms and then spending (Ashworth and Bueno de Mesquita (2009); Zakharov (2009)), spending and then platforms (Serra (2010)), or choosing in a predetermined order (Wiseman (2006)).¹

¹For brevity do not detail other substantial related literatures that assume either platforms *or* valence are exogenous,

In the real world candidates do not literally choose all strategies simultaneously; however, since they can both adjust platforms and spending throughout a campaign (Iaryczower, Lopez and Meirowitz (2017)), simultaneity is an arguably more agnostic assumption than any particular fixed sequence. A second key difference is that our candidates have a mixture of policy and office motivations; this allows us to identify important differences between them.

Several of our results appear in these other works, but sometimes for different reasons. In Zakharov (2009) and Ashworth and Bueno de Mesquita (2009) office-motivated candidates uncertain about the median’s preferences diverge to dampen the effect of spending and intensity of competition; but this effect is sensitive to the form of candidate uncertainty. Incentives in our model are more similar to Wiseman (2006) and Serra (2010); the latter also finds a positive association between polarization and spending, and a negative one between both and the cost of valence. Key features of that model’s equilibrium, however, do not fit the data well. Voters and candidates know ex-ante where candidates will position and who win, while real elections exhibit substantial uncertainty. The losing candidate always converges to the median, while real-world elections exhibit substantial divergence by both parties (Bafumi and Herron (2010)). And polarization is a function of the *asymmetry* in candidate extremism – as it shrinks, both polarization and the comparative statics effects vanish.

While our paper clearly complements rather than substitutes for existing work, one clear advantage is its tractability, which makes it well suited for empirical testing. Despite the apparent difficulty of working with mixed strategies, it has a unique, continuous, easily expressed equilibrium, with clear comparative statics in all parameters and outcomes, across the full parameter space. Previous works exhibit complex and very discontinuous strategies as well as equilibrium multiplicity (Ashworth and Bueno de Mesquita (2009)), only explore part of the parameter space (Serra (2010)), do not solve for Nash equilibria (Zakharov (2009)), or require numerical derivations (Wiseman (2006)); several conduct comparative statics analysis via examples (Wiseman (2006), Ashworth and Bueno de Mesquita (2009)).

The Model

Two candidates $i \in \{-1, 1\}$ simultaneously choose ideological platforms $\gamma_i \in \mathbb{R}$ and costly spending levels $q_i \in [0, \infty) = \mathbb{R}^+$. A median voter (V) then votes for her preferred candidate and the game

or that endogenize both but include other distinctive features – see Zakharov (2009) for a review.

ends. Platforms are commitments to spatial policies that will be implemented in office. Spending is a reduced-form representation for costly actions that make voting for a given candidate more appealing to the voters, holding platforms fixed. The median’s ideal is normalized to 0, and her utility for selecting candidate i is $\mu q_i - \lambda_v \gamma_i^2$. She thus places a weight λ_v on ideological outcomes, and a candidate spending q_i generates a *valence* return of μq_i . For now we are agnostic about the interpretation of valence. Interpreted literally, valence makes the voters better off. However, valence could instead proxy for how spending biases voters without increasing welfare (e.g. uninformative campaigning).² μ reflects the *sensitivity* of the voter’s valence perception to a dollar of campaign spending. Note that the voters’ preferences are known to the candidates, in contrast to Calvert (1985).

Candidates have both policy and office motivations. Letting w denote the election winner (and γ_w the winning platform), candidate i ’s final utility from the election outcome is $\mathbf{1}_{i=w}\theta - \lambda_c (x_i - \gamma_w)^2$. x_i is candidate i ’s ideal ideology. As in the canonical model by Calvert (1985), candidates have policy goals, but must first achieve the “proximate” goal of election to achieve those goals. θ represents office-holding benefits such as ego rents or salary. λ_c is the candidates’ weight on ideology. Candidates prefer to avoid spending, which costs $a q_i$ and enters additively into their utility. We consider candidates who are equidistant from the median ($|x_i| = |x_{-i}| = x$) on opposite sides, so candidate i ’s ideal is $x_i = ix$.

Equilibrium

The model is a variant of an all-pay contest (Siegel (2009)) and related to the policy development model of Hirsch and Shotts (2015).³ A candidate’s strategy is a two-dimensional “bid” (γ_i, q_i) consisting of a platform and spending level in a “contest” to win the median’s support. The median’s utility $s_i = \mu q_i - \lambda_v \gamma_i^2$ for candidate i is the “score” of candidate i ’s “bid,” meaning it is the quantity determining who wins. A candidate can increase her score in two ways. First, she can increase spending – this is costly up front, but has no effect on the benefit from winning. Second, she can position closer to the median – this is “free” up front, but makes winning less valuable.

There is a unique equilibrium that is in symmetric mixed strategies; candidates randomize over

²Spending may also represent the sum of positive campaigning to improve a candidates’ own image and negative campaigning to worsen her opponent’s image – see Appendix D for details.

³Differences in the current model are: (i) the median voter has no “outside option” to the two candidates, (ii) candidates care about winning and not just policy, (iii) “valence” is only valued by the median and not also the candidates, (iv) the median’s weight on valence may vary, and (v) the ideological intensity of both candidates and voters may vary.

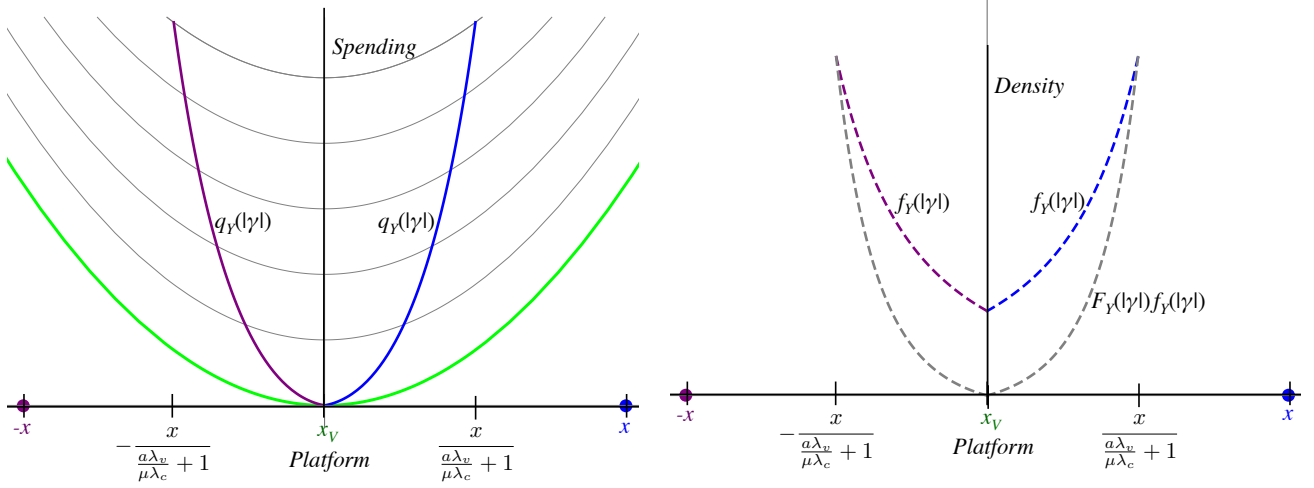


Figure 1: *Equilibrium Strategies and Outcomes*

both platforms and spending. Candidates must randomize for similar reasons as a classic all-pay contest; a candidate who is spending something but losing for sure has an incentive to instead spend nothing, a candidate who is winning by a strictly positive margin has a strict incentive to spend less, and candidates who are tied both have an incentive to spend a little bit more and win.⁴

Proposition 1. *The probability that a candidate positions closer to the median than distance y is $F_Y(y) = \min \left\{ \left(\frac{a\lambda_v}{\mu\lambda_c} \right) \left(\frac{x}{x-y} \right), 1 \right\}$. When positioning at distance y , candidate i selects platform $\gamma_i(y) = iy$ and spends $q_Y(y) = \frac{1}{\mu} (s_Y(y) + \lambda_v y^2)$, where $s_Y(y) = \int_0^y \lambda_v \frac{x}{(x-\hat{y})^2} \left(\frac{\theta}{\lambda_c} + 4x\hat{y} \right) d\hat{y}$.*

Strategies are depicted in Figure 1. The left panel shows the platforms (on the x-axis) and spending (on the y-axis) that the left (purple) and right (blue) candidate mix over. The median's indifference curves, i.e., the pairs of platforms and spending that she is equally willing to vote for, are in gray. The right panel shows the density (PDF) over the left (purple) and right (blue) candidates' platforms, as well as the density (in gray) of the final policy outcome (i.e., the platform of the winning candidate).⁵

Properties of Equilibrium

Uncertainty Although the game is of complete and perfect information, in the unique equilibrium both candidates and voters are uncertain about where candidates will position, how much they will

⁴Proving this is more involved than the all pay contest because payoffs from winning and losing are endogenous.

⁵See Appendix D for details on calculating the distribution over the final policy outcome.

spend, and who will win. This unpredictability is a unique feature of the model that arises from the candidates’ need to remain electorally competitive in the presence of campaign spending.

Divergence The unique equilibrium exhibits divergence – both candidates position away from the median with probability 1 ($F_Y(0) = 0$). The reason is similar to the classic Calvert model, but that model requires candidates to be uncertain about the median’s preferences. This induces them to gamble on sometimes winning with a divergent platform. In our model both the median’s preferences and the effect of spending are known. However, the *strategic* uncertainty produced by campaign spending, and the resulting uncertainty in platforms, serves a similar role – platforms away from the median’s ideal may still win the election, and are therefore sometimes adopted.

Inefficiency The unique equilibrium is inefficient in both spending and policy outcomes. Spending is inefficient in two senses. First, it is pure waste if assumed to only affect the voters “behaviorally.” Second, the losing candidate always wastes money trying to win. Equilibrium policy outcomes are efficient “on average” because they are centered on the median. However, they are also uncertain ex-ante, harming both candidates and voters due to risk aversion. The root of these inefficiencies is the nature of campaign spending itself. Spending allows the candidates to bias policy outcomes away from the median. However, unlike a promised transfer in exchange for a voters’ support, a candidate must pay for campaign spending *before* she knows whether it will yield an electoral victory.

Polarization The unique equilibrium exhibits *polarization* in candidate positions – each candidate is *more likely* to take extreme positions closer to her own ideal point than moderate ones closer to the median ($f_Y(y) > 0$ in the support), and places vanishing weight on platforms near the median. The reason is that candidates are risk averse over policy, so the marginal benefit of “buying” support for a more extreme platform with spending is largest for platforms near the median.

Voter Extremism In equilibrium candidates spend more, and have higher valence, when taking extreme platforms ($q'_Y(y) > 0$). This differs with models of exogenous valence, where high-valence candidates position near the median to have their advantage overwhelm their opponent (see [Stone and Simas \(2010\)](#) p. 373 for a review). This difference is partially due to the endogeneity of valence – candidates choose to invest in valence in order *to allow* positioning away from the median.

More surprising is that candidates spend *so much* more when taking extreme platforms that the

median actually evaluates their overall candidacy more favorably. (In the left panel of Figure 1 the candidates' spending functions are steeper than the median's indifference curves). The median thus always votes for the most *extreme* candidate, and appears to have a preference for extremism! Consequently, policy outcomes (i.e., the platform of the winning candidate) are even more polarized than the policy platforms (see the right panel of Figure 1). The reason for this counterintuitive effect is as follows. To win, candidates trade off spending against ideological concessions. When a candidate aims to be more competitive (that is, more likely to win), ideological concessions become costlier because the platform is more likely to actually be implemented. Reversing the statement, a strategy that makes fewer ideological concessions (i.e. that is more extreme) must also be more likely to win.

Comparative Statics

We last analyze how changes in the model parameters affect various outcomes. Because strategies are uncertain, this involves analyzing *first-order stochastic* changes in the distribution of these outcomes. Recall that a candidate's spending is q_i while her valence is μq_i ; we analyze the median voter's equilibrium welfare both including valence μq_i (i.e., interpreting valence literally as a productive attribute) and excluding it (i.e., interpreting valence "behaviorally" as a bias).

Proposition 2. *The six model parameters (in columns) first order stochastically increase (+), decrease (-), or have no effect on (0), the six equilibrium outcome quantities (in rows).⁶*

	x	λ_c	θ	a	λ_v	μ
Platform Extremism	+	+	0	-	-	+
Spending	+	+	+	-	-*	+
Valence	+	+	+	-	-*	+
Candidate Welfare	-	-	0	+	+	-
Voter Welfare (Behavioral)	-	-	0	+	+	-
Voter Welfare	+	+	+	-	-	+

Platforms, Spending, and Valence The model predicts a positive association between platform polarization and spending. The reason is simple: polarization reflects the candidates' willingness to spend to gain support for more extreme platforms. To interpret how the model parameters influence this willingness (and thus both polarization and spending), we divide them into three categories.

⁶Starred results further require $\frac{a}{\mu} \geq \frac{\lambda_c}{\lambda_v}$. This states that the marginal cost generating valence – that is, of influencing voters via spending – is sufficiently high, and specifically exceeds the candidates' (relative) valuation of ideology. When this condition fails, the indicated parameters do not cause a first-order stochastic change in the indicated outcomes.

The first is the candidates’ *desire for extreme policies*, determined by their ideological strength λ_v and extremism x . Unsurprisingly, increasing the candidates’ desire for extreme policies magnifies their willingness to spend to gain support for them, generating both more extreme platforms and greater spending and valence. The second is the candidates’ *ability* to translate spending into support for extreme policies, jointly determined by (i) the candidates’ marginal cost a of spending, (ii) the effectiveness μ of a dollar of spending on valence, and (iii) the median’s ideological strength λ_v , which determines how much spending is needed to “compensate” her for extreme platforms. Increasing ability through any of these channels also generates more extreme platforms and polarization. The third is the candidates’ nonideological officeholding benefits θ . Higher officeholding benefits increase spending and valence, but surprisingly, have *no effect* on candidate platforms despite the greater benefit to winning. The reason is that the level of officeholding benefits does not influence the calculus of using spending to gain support for more extreme platforms, which is what drives polarization.

Welfare Candidates spend to both pull policy toward their ideal when they win, and to prevent losing to something ideologically unappealing. On average, however, they do not move outcomes from the median, and waste money and generate policy uncertainty trying to. They are thus clearly harmed by spending competition; the degree of platform polarization indicates by how much. That is, they become worse off when their own desire for extreme policies (higher x or λ_c), or ability to translate spending into support (lower a , higher μ , or lower λ_v), go up. (They are unaffected by changes in officeholding benefits θ because they compete all these away through campaign spending.)

Whether the voter benefits from or is harmed by spending competition depends on whether her “revealed preference” for spending is a behavioral bias or a true reflection of greater welfare, confirming a conjecture in [Serra \(2010\)](#). With the former assumption the median is harmed; the degree of platform polarization indicates by how much. With the latter assumption the median benefits, and the level of *spending* indicates by how much. That is, she becomes better off when the candidates become more extreme (higher x), or are better able to translate spending into valence (higher μ), even though platforms also polarize. Moreover, she would be harmed by constraints or a ban on fundraising and spending (which also sometimes holds in [Ashworth and Bueno de Mesquita \(2009\)](#)).

Discussion and Conclusion

We conclude with two issues we believe are worthy of greater consideration in light of our results. First, a growing literature estimates the electoral penalty to extremism by examining the empirical relationship between candidate positions and citizens' votes (e.g. [Tausanovitch and Warshaw \(2018\)](#)). Our model shows that failing to account for strategic positioning can introduce dramatic bias into such studies – in equilibrium, there will appear to be an electoral *reward* to extreme positions, because candidates spend so much more when choosing them so as to overwhelm the true electoral penalty. Indeed, recent empirical work finds support for the proposition that candidates position strategically ([Iaryczower, Lopez and Meirowitz \(2017\)](#)), and studies accounting for strategic positioning when estimating the electoral penalty (e.g. [Hall \(2015\)](#)) yield very different results from those that do not.

The second issue is the difficulty of reforming institutions with the intent of reducing polarization. [Hall \(2018\)](#) plausibly argues that polarization has risen in part due to a devaluation of public service, and proposes making officeholding more valuable to select for less ideologically-motivated candidates. Our model illustrates the weaknesses of such a simple antidote. Because US elections are costly and competitive, candidates may simply compete away any new benefits of officeholding; leaving unchanged both the value of moderation to extreme candidates, and the value of candidacy to moderate ones.

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Online Appendix

This Appendix has four parts. Appendix [A](#) informally derives necessary equilibrium conditions. Appendix [B](#) uses these conditions to derive the unique equilibrium; this subsumes the characterization in Proposition 1 in the main text. Appendix [C](#) proves the comparative statics results stated in Proposition 2 in the main text. Finally, Appendix [D](#) rigorously proves the necessary equilibrium conditions stated in Appendix [A](#) as well as two other propositions used in Appendix [B](#), and explains two footnotes in the main text.

A Informal Derivation of Necessary Conditions

The game is a multidimensional contest in which a candidate’s “proposal” (γ, q) is her combination of platform and spending, and the “scoring rule” applied to proposals is just the median voter’s utility $\mu q - \lambda_v \gamma^2$, since she decides which candidate to elect. We henceforth refer to candidate strategies as proposals. We also sometimes refer to the median voter as the decisionmaker (DM). To facilitate the analysis we reparameterize proposals (γ, q) to be expressed in terms of (s, γ) , where $s = \mu q - \lambda_v \gamma^2$ is the DM’s utility for a proposal or its *score*. The implied spending on a proposal (s, γ) is then $q = \frac{1}{\mu} (s + \lambda_v \gamma^2)$ which costs $\alpha (s + \lambda_v \gamma^2)$ where $\alpha = \frac{a}{\mu}$ is the cost to the candidate of generating one unit of valence. Note that the DM’s ideal point with no spending has exactly 0 score, and is the most competitive “free” proposal to make.

In the reparameterized game, a candidate’s pure strategy (s_i, γ_i) is a two-dimensional element of $\mathbb{B} \equiv \{(s, \gamma) \in \mathbb{R}^2 \mid s + \gamma^2 \geq 0\}$. A mixed strategy σ_i is a probability measure over the Borel subsets of \mathbb{B} . Let $F_i(s)$ denote the CDF over scores induced by i ’s mixed strategy σ_i . For technical convenience we restrict attention to strategies generating score CDFs that can be written as the sum of an absolutely continuous and a discrete distribution.

In this section we informally derive necessary conditions for equilibrium by assuming that the strategies satisfy the following intuitive and simplifying conditions. First, each candidate uses an absolutely continuous and atomless score CDF $F_i(s_i)$ with support over a common interval of scores $[0, \bar{s}]$. Second, each candidate positions at a unique platform $\gamma_i^S(s_i)$ at each

score in the support. Finally, at most one candidate has an atom at score 0 ($F_k(0) > 0$ for at most one $k \in \{L, R\}$). In Appendix D we discard these assumptions and rigorously prove the stated necessary conditions.

Optimal Platforms

With our assumptions, a candidate's utility from making proposal (s_i, γ_i) is

$$-\alpha (s_i + \lambda_v \gamma_i^2) + F_{-i}(s_i) \cdot (\theta - \lambda_c (ix - \gamma_i)^2) + \int_{s_{-i} > s_i} -\lambda_c (ix - \gamma_{-i}^S(s_{-i}))^2 d\sigma_{-i}. \quad (\text{A.1})$$

The first term is the up-front cost of spending. The second term is the probability i 's proposal wins the election times her utility for it, which includes both policy losses $-\lambda_c (ix - \gamma_i)^2$ and office-holding benefits θ . The third term is i 's utility should she lose, which requires integrating over all her opponent's proposals at potentially different platforms with higher scores than s_i . Taking the derivative with respect to γ_i and setting equal to 0 yields that i 's optimal platform $\gamma_i^S(s_i)$ at each score must be equal to

$$\gamma_i^S(s_i) = i \left(\frac{F_{-i}(s_i)}{\Lambda\alpha + F_{-i}(s_i)} \right) x$$

where $\Lambda = \frac{\lambda_v}{\lambda_c}$. Thus, optimal platform at a score s_i becomes more extreme the further are the candidates from the DM, and more moderate as the cost α of generating a unit of valence or the DM's relative ideological strength $\Lambda = \frac{\lambda_v}{\lambda_c}$ increases.

The optimal platform may also be written as $\gamma_i^S(s_i) = \gamma_i(y(F_{-i}(s_i)))$, where $\gamma_i(y) = iy$ and $y(P) = \frac{P}{\Lambda\alpha + P}x$. Doing so clarifies two properties. First, the platform is fully characterized by its *distance* $y(F_{-i}(s_i))$ from the DM, since a candidate will only position on her side of the DM. Second, the optimal distance only depends on the score s_i through the *probability* $F_{-i}(s_i)$ that her opponent $-i$ makes a lower score proposal than s_i . Since $y(P)$ is strictly increasing in P , this means that i 's optimal platform is more distant from the DM the greater is the targetted score s_i and by implication the more likely she is to win.

Equilibrium Score Conditions

To derive the equilibrium score CDFs (F_i, F_{-i}) , note that every score $s_i \in [0, \bar{s}]$ in the CDFs' common support must maximize i 's utility when the optimal platforms $\gamma_i^S(s_i)$ are chosen. Since i is indifferent over all scores in $[0, \bar{s}]$, substituting $\gamma_i^S(s_i)$ into equation A.1,

differentiating with respect to s_i and setting it equal to zero over $[0, \bar{s}]$ yields a pair of differential equations that must be jointly satisfied in equilibrium:

$$f_{-i}(s) \cdot (\theta + \lambda_c ((x + y(F_i(s)))^2 - (x - y(F_{-i}(s)))^2)) = \alpha \quad \forall i \text{ and } s \in [0, \bar{s}] \quad (\text{A.2})$$

Equation A.2 has a natural interpretation. The right side is i 's cost of increasing her own valence, which increases her score. The left side is i 's marginal ideological gain from increasing her score: with “probability” $f_{-i}(s)$ she goes from losing to winning the election, which shifts the outcome from an ideology that is distance $y(F_i(s))$ from the DM on the *opposite* side, to an ideology that is distance $y(F_{-i}(s))$ from the DM on the *same* side.

Necessary Conditions

Given the above, necessary conditions for SPNE are as follows. (A rigorous proof that discards the simplifying assumptions is in Appendix D).

Proposition A.1. *Necessary conditions for SPNE are as follows:*

1. (**Ideological Optimality**) *With probability 1, proposals take the form $(s_i, \gamma_i(y(F_{-i}(s))))$, where $\gamma_i(y) = iy$, $y(P) = \frac{P}{\Lambda\alpha + P}x$, and $\Lambda = \frac{\lambda_v}{\lambda_c}$.*
2. (**Score Optimality**) *Score CDFs (F_i, F_{-i}) must satisfy $F_k(0) > 0$ for at most one k , $\text{supp}\{F_i\} \cap [0, \infty] = \text{supp}\{F_{-i}\} \cap [0, \infty] = [0, \bar{s}]$ where $\bar{s} \in (0, \infty]$, and*

$$f_{-i}(s) \cdot (\theta + \lambda_c ((x + y(F_i(s)))^2 - (x - y(F_{-i}(s)))^2)) = \alpha \quad \forall i \text{ and } s \in [0, \bar{s}]$$

B Derivation of Equilibrium Strategies

Using the necessary conditions stated in Proposition A.1 we now explicitly derive the unique equilibrium. We first use that any pair of equilibrium score CDFs must be identical, i.e., $F_i(s) = F_{-i}(s) = F(s)$, implying that any equilibrium must be symmetric and unique (proof in Appendix D).

Proposition B.1. *In any SPNE, $F_i(s) = F_{-i}(s) = F(s)$, where $F(0) = 0$ and*

$$f(s) (\theta + 4\lambda_c xy(F(s))) = \alpha \quad \forall s \in [0, \bar{s}]$$

Using this we can explicitly derive the *inverse* of the unique score CDF $F(s)$ satisfying Proposition B.1, which we denote as $s_F(P)$. Since $F(0) = 0$, the inverse straightforwardly satisfies the boundary condition $s_F(0) = 0$. Now using that $f(s) = \frac{1}{s'_F(F(s))}$ and that $y(P) = \frac{Px}{\Lambda\alpha + P}$, the differential equation may be rewritten as

$$s'_F(P) = \frac{1}{\alpha} \left(\theta + 4\lambda_c x \frac{Px}{\Lambda\alpha + P} \right) \quad \forall P \in [0, 1] \quad (\text{B.1})$$

Finally $s_F(P) = s_F(P) - s_F(0) = \int_0^P s'_F(\hat{P}) d\hat{P}$; the first equality follows from the boundary condition, and the second from the fundamental theorem of calculus.

The preceding characterizes the unique strategies satisfying Proposition A.1 as a common score CDF $F(s)$ that the candidates mix over, and the platform $\gamma_i(y(F(s)))$ at which candidate i positions when generating score s (implying a spending level of $q_S(s) = \frac{s + \lambda_v [\gamma_i(y(F(s)))^2]}{\mu}$). However, these strategies may be expressed equivalently but more intuitively as a CDF $F_Y(y)$ over the *distance* of each candidate's platform from the DM (that is, their extremism), and a platform $\gamma_i(y)$ and level of spending $q_Y(y)$ when positioning distance y from the DM.

To derive these quantities from the original characterization, we first argue that $y_F(P)$ (the inverse of $F_Y(y)$) is exactly equal to $y(P)$ as defined in Proposition A.1, and hence

$$F_Y(y) = \alpha\Lambda \left(\frac{y}{x - y} \right).$$

First, since there is a positive association between platform extremism and score, $F_Y(y) = F(s_Y(y))$, where $s_Y(y)$ denotes the score of a candidate's platform when positioning distance y from the DM. This in turn implies that $y_F(P) = y_S(s_F(P))$. Second, letting $y_S(s)$ denote the inverse of $s_Y(y)$, by Proposition A.1 we have $y_S(s) = y(F(s)) \rightarrow y_S(s_F(P)) = y(P)$.

Next we derive $s_Y(y)$, which implies spending $q_Y(y) = \frac{1}{\mu}(s_Y(y) + \lambda_v y^2)$. Recall that $s_Y(y)$ is just the inverse of $y(F(s))$. We then have that $s_Y(y) = s_F(F_Y(y))$, recalling that $s_F(P)$ denotes the inverse of $F(s)$. Now differentiating both sides yields that $s'_Y(y) = s'_F(F_Y(y)) f_Y(y)$, and it is easily derived that $f_Y(y) = \frac{\alpha\Lambda x}{(x-y)^2}$. Substituting in then yields

$$s'_Y(y) = \frac{x}{(x-y)^2} \Lambda (\theta + 4\lambda_c xy) \quad (\text{B.2})$$

Finally, $F(0) = 0$ and $y(0) = 0 \rightarrow y(F(0)) = 0 \rightarrow s_Y(y) = 0$ (that is, when positioning at the DM's ideal a candidate's proposal has 0-score). Thus $s_Y(y) = s_Y(y) - s_Y(0) = \int_0^y s'_Y(y) dy$, where again the first equality comes from the boundary condition and the

second from the fundamental theorem of calculus.

The following proposition both summarizes the preceding derivations and proves the derived strategies are sufficient as well as necessary for equilibrium (subsuming Proposition 1 in the main text). It is proved in Appendix D.

Proposition B.2. *There is a unique SPNE. The equilibrium is symmetric and can be equivalently described as follows.*

1. *Each candidate chooses the score of her proposal according to a common CDF $F(s)$, and positions at platform $\gamma_i(y(F(s))) = i \frac{F(s)}{\Lambda\alpha + F(s)}x$ when making a proposal with score s . The inverse $s_F(P)$ of $F(s)$ is equal to*

$$s_F(P) = \int_0^P \frac{1}{\alpha} \left(\theta + 4\lambda_c x \frac{\hat{P}x}{\Lambda\alpha + \hat{P}} \right) d\hat{P},$$

2. *Each candidate chooses the distance of her platform from the median voter according to a common CDF $F_Y(y) = \alpha\Lambda \left(\frac{y}{x-y} \right)$. When positioning at distance y , candidate i selects platform $\gamma_i(y) = iy$ and spends $q_Y(y) = \frac{1}{\mu} (s_Y(y) + \lambda_v y^2)$, where*

$$s_Y(y) = \int_0^y \frac{x}{(x-\hat{y})^2} \Lambda (\theta + 4\lambda_c x \hat{y}) d\hat{y},$$

C Comparative Statics

We now prove the comparative statics results in Proposition 2. For the proofs we consider each quantity of interest separately.

C.1 Probabilistic Outcomes

Because the candidates mix over continuum of platforms and spending levels, most of the equilibrium outcomes we consider are probabilistic; in particular, the candidates' platforms, spending, valence, and the DM's utility. To analyze comparative statics we thus consider *first-order* stochastic changes in the outcome of interest (when such comparisons are possible). Recall that for two distributions over a univariate outcome z described by CDFs $F_Z(z)$ and $\hat{F}_Z(z)$, distribution F_Z first-order stochastically dominates distribution \hat{F}_Z if and only if $F_Z(z) \leq \hat{F}_Z(z) \forall z$ and $F_Z(z) < \hat{F}_Z(z)$ for some z .

For the subsequent proofs we rely on the following straightforward observations. Consider an absolutely continuous CDF $F_Z(z; m)$ describing the distribution over some outcome z , and which is also a continuously differentiable function of some parameter m . Further suppose the CDF satisfies $F_Z(0; m) = 0 \forall m$ and $F_Z(\bar{z}(m); m) = 1$ for some $\bar{z}(m) > 0$. Then the CDF has a well-defined inverse $z_F(P; m)$ over $P \in [0, 1]$ satisfying $z(0; m) = 0$ and $z(1; m) = \bar{z}(m)$. In addition, $\frac{\partial F_Z(z; m)}{\partial m} < 0 \forall z \in (0, \bar{z}(m))$ or $\frac{\partial z_F(P; m)}{\partial m} > 0 \forall P \in (0, 1)$ equivalently imply that the distribution is first-order stochastically increasing in m (that is, $F_Z(z; m')$ first-order stochastically dominates $F_Z(z; m)$ for $m' > m$), while the reverse signs imply that it is first-order stochastically decreasing in m .

We first consider the distribution over the candidates' platform extremism.

Proposition C.1. *The candidates' platform extremism is first-order stochastically increasing in (x, λ_c, μ) , decreasing in (a, λ_v) , and unaffected by θ .*

Proof: The CDF over each candidates' platform extremism $F_Y(y) = \alpha \Lambda \left(\frac{y}{x-y} \right)$, which is transparently first-order stochastically decreasing in $\alpha = \frac{a}{\mu}$ and $\Lambda = \frac{\lambda_v}{\lambda_c}$, increasing in x , and unaffected by θ . QED

We next consider comparative statics in the score CDF $F(s)$. This is necessary as an intermediate step to analyze statics in spending and valence, as well as the DM's equilibrium utility if valence is interpreted literally.

Proposition C.2. *The score of the candidates' platforms is first-order stochastically increasing in $(x, \lambda_c, \mu, \theta)$, and decreasing in (a, λ_v) .*

Proof: It is simpler to work with the inverse $s_F(P) = \int_0^P \frac{1}{\alpha} \left(\theta + 4\lambda_c x \frac{\hat{P}x}{\Lambda\alpha + \hat{P}} \right) d\hat{P}$ for $P \in (0, 1)$ and differentiate under the integral sign (since the model parameters do not enter the limits of integration). Doing so transparently yields that function and hence the distribution is decreasing in $\alpha = \frac{a}{\mu}$ and λ_v and increasing in θ , x , and λ_c . QED

We next consider comparative statics in the valence generated by each candidate.

Proposition C.3. *A candidate's valence is first-order stochastically increasing in $(x, \lambda_c, \mu, \theta)$ and decreasing in a . When $\frac{a}{\mu} \geq \frac{\lambda_c}{\lambda_v} \iff \alpha \Lambda \geq 1$, it is also decreasing in λ_v .*

Proof: A candidate with platform (s, y) spends $q = \frac{1}{\mu} (s + \lambda_v y^2)$ and thus has valence $v = \mu q = s + \lambda_v y^2$. Now letting $F_V(v)$ denote the CDF over a candidates' valence, it is straightforward to see that this relationship implies that the inverse of $F_V(v)$ is simply

$v_F(P) = s_F(P) + \lambda_v [y_F(P)]^2$. Results about the effect of α , θ , x , and λ_c are then immediately implied by Propositions C.1 and C.2; $v_F(P)$ is strictly increasing in both $s_F(P)$ and $y_F(P)$, these parameters only influence $v_F(P)$ through $s_F(P)$ and $y_F(P)$, and their impact on these two functions have (weakly) the same sign.

To evaluate the effect of λ_v on $v_F(P)$ it is straightforward that $\frac{\partial}{\partial \lambda_v} (\lambda_v [y_F(P)]^2) < 0$ $\forall P \in (0, 1]$ is a *sufficient* (but not necessary) condition for $v_F(P)$ to be decreasing in λ_v (in conjunction with Proposition C.2) and hence the desired result. Now $\lambda_v [y_F(P)]^2 = \lambda_v \left(\frac{P}{\Lambda\alpha + P}x\right)^2 = \frac{x^2\lambda_c}{\alpha} \cdot \Lambda\alpha \left(\frac{P}{\Lambda\alpha + P}\right)^2$. Since $\Lambda\alpha$ is strictly increasing in λ_v it suffices to show that $\frac{\partial}{\partial(\Lambda\alpha)} \left(\Lambda\alpha \left(\frac{P}{\Lambda\alpha + P}\right)^2\right) < 0$ $\forall P \in (0, 1)$ which in turn is implied by $\frac{\partial}{\partial(\Lambda\alpha)} \left(\log \left(\Lambda\alpha \left(\frac{P}{\Lambda\alpha + P}\right)^2\right)\right) = -\frac{\Lambda\alpha - P}{\Lambda\alpha(\Lambda\alpha + P)} < 0$ (since log of a strictly positive number is a strictly increasing transformation with finite derivative), which is transparently the case $\forall P \in (0, 1)$ when $\Lambda\alpha \geq 1$. QED

We next consider comparative statics in the spending by each candidate.

Proposition C.4. *A candidate's spending is first-order stochastically increasing in (x, λ_c, θ) and decreasing in a . When $\alpha\Lambda \geq 1$ it is also increasing in μ and decreasing in λ_v .*

Proof: A candidate with platform (s, y) spends $q = \frac{1}{\mu} (s + \lambda_v y^2)$. Letting $F_Q(q)$ denote the CDF over a candidates' spending, the preceding relationship implies that the inverse is $q_F(P) = \frac{1}{\mu} (s_F(P) + \lambda_v [y_F(P)]^2)$. Now it is clear that x, λ_c, θ, a , and λ_v only impact spending through valence $s_F(P) + \lambda_v [y_F(P)]^2$; hence the stated comparative statics are implied by Proposition C.3.

To evaluate the impact of μ , observe that

$$\begin{aligned} q_F(P) &= \frac{1}{\mu} \left(\int_0^P \frac{1}{\alpha} \left(\theta + 4\lambda_c x \frac{\hat{P}x}{\Lambda\alpha + \hat{P}} \right) d\hat{P} + \lambda_v [y_F(P)]^2 \right) \\ &= \int_0^P \frac{1}{a} \left(\theta + 4\lambda_c x \frac{\hat{P}x}{\Lambda\alpha + \hat{P}} \right) d\hat{P} + \frac{\lambda_c}{a} \Lambda\alpha \left[\frac{P}{\Lambda\alpha + P} x \right]^2 \end{aligned}$$

Recalling that $\alpha = \frac{a}{\mu}$, the first term is clearly increasing in μ . To see the second term is increasing in μ , observe that since $\Lambda\alpha$ is decreasing in μ it suffices to show that $\frac{\partial}{\partial(\Lambda\alpha)} \left(\Lambda\alpha \left(\frac{P}{\Lambda\alpha + P}\right)^2\right) < 0$ $\forall P \in (0, P)$ when $\alpha\Lambda \geq 1$, which is already shown in the proof of Proposition C.3. QED.

We last consider comparative statics in the DM's utility. We first analyze these statics when valence is interpreted "literally."

Proposition C.5. *If valence is interpreted literally, then the median voter’s equilibrium utility is first-order stochastically increasing in $(x, \lambda_c, \mu, \theta)$, and decreasing in (a, λ_v) . It thus tracks the amount of spending (in the sense of obeying identical comparative statics with respect to all six individual model parameters) when $\alpha\Lambda \geq 1$.*

Proof: If valence is interpreted literally, the DM’s equilibrium utility is equal to the score of the candidate she selects. Since she always selects the highest score, the distribution over her equilibrium utility is described by the CDF over the maximum score $[F(s)]^2$, and thus obeys identical comparative statics as the score CDF (characterized in Proposition C.2). Finally, by Proposition C.4 candidate spending also obeys the same comparative statics when $\alpha\Lambda \geq 1$. QED

We now analyze these statics when valence is interpreted “behaviorally” and excluded from the welfare calculation.

Proposition C.6. *If valence is interpreted behaviorally, the median voter’s equilibrium utility is first-order stochastically decreasing in $(x, \lambda_c, \mu, \theta)$ and increasing in a . When $\alpha\Lambda \geq 1$ it is also increasing in λ_v and thus inversely tracks the extremism of platforms (in the sense of obeying opposite comparative statics with respect to all six individual model parameters).*

Proof: If valence is interpreted behaviorally, then the DM’s utility for a proposal (s_i, y_i) is $-\lambda_v y_i^2$. Let $\ell_i = \lambda_v y_i^2$ denote the DM’s losses from the proposal (so losses are the negative of utility), and let $F_L(\ell)$ denote the CDF over losses that the candidates’ platforms offer the DM. Since the DM (incorrectly) chooses the candidate with the most extreme platform due to spending, she always suffers the maximum losses presented to her, and thus the CDF over her behavioral losses is $[F_L(\ell)]^2$. By implication, first-order stochastic increases (decreases) in the losses offered by the candidates’ platforms imply corresponding first-order stochastic increases (decreases) in the DM’s equilibrium losses, and first-order stochastic decreases (increases) in the DM’s (behavioral) equilibrium utility. Finally, the inverse $\ell_F(P)$ of a candidate’s loss CDF $F_L(\ell)$ is just $\lambda_v [y_F(P)]^2$. The parameters $(x, \lambda_c, \mu, \theta, a)$ thus only affect behavioral losses through the extremism of platforms, and so each of these parameters affect platform extremism and equilibrium losses in the same way. For the parameter λ_v , equilibrium losses are first order stochastically decreasing for $\alpha\Lambda \geq 1$ when $\frac{\partial}{\partial \lambda_v} (\lambda_v [y_F(P)]^2) < 0$, which is already shown in the proof of Proposition C.3. QED

C.2 Candidate Utility

The candidates' equilibrium utility is the only outcome of interest that is not probabilistic, since mixing implies that the candidates achieve their exact equilibrium utility with any proposal in the support of their strategy. In particular, candidate i achieves her equilibrium utility with proposal $\left(\bar{s}, i\left(\frac{F(\bar{s})}{\Lambda\alpha + F(\bar{s})}\right)x\right) = (s_F(1), i\left(\frac{1}{\Lambda\alpha + 1}\right)x)$; that is, a proposal with the maximum score (which ensures winning the election for sure) alongside her optimal platform if she expects to win for sure. This yields the following comparative statics in the candidates' equilibrium utility.

Proposition C.7. *The candidates' equilibrium utility is decreasing in (x, λ_c, μ) , increasing in (a, λ_v) , and unaffected by θ . It thus inversely tracks the extremism of platforms.*

Proof: A candidates' utility from platform $(s_F(1), i\left(\frac{1}{\Lambda\alpha + 1}\right)x)$ and thus her equilibrium utility is equal to

$$\begin{aligned} & -\alpha \left(s_F(1) + \lambda_v \left(\frac{x}{\Lambda\alpha + 1} \right)^2 \right) + \left(\theta - \lambda_c \left(x - \left(\frac{x}{\Lambda\alpha + 1} \right) \right)^2 \right) \\ & = \theta - \alpha s_F(1) - x^2 \lambda_c \frac{\Lambda\alpha}{\Lambda\alpha + 1} = -x^2 \lambda_c \left(\int_0^1 \frac{4\hat{P}}{\Lambda\alpha + \hat{P}} d\hat{P} + \frac{\Lambda\alpha}{\Lambda\alpha + 1} \right) \end{aligned}$$

We now make three straightforward observations. First, candidate utility is clearly decreasing in x and unaffected by θ . Second, the effect of (a, μ, λ_v) is mediated exclusively through $\Lambda\alpha$, which only affects the term inside the parentheses. Third, a sufficient condition for the utility to be decreasing in λ_c is that the term inside the parentheses is increasing in λ_c .

Finally, taking the derivative of this term inside the parentheses w.r.t. $\Lambda\alpha$ yields

$$-\int_0^1 \frac{4\hat{P}}{(\Lambda\alpha + \hat{P})^2} d\hat{P} + \frac{1}{(\Lambda\alpha + 1)^2} < -\int_0^1 \frac{4\hat{P}}{(\Lambda\alpha + 1)^2} d\hat{P} + \frac{1}{(\Lambda\alpha + 1)^2} = -\frac{1}{(\Lambda\alpha + 1)^2} < 0.$$

This proves that utility is increasing in $\Lambda\alpha$ (holding λ_c fixed) and so increasing in (a, λ_v) and decreasing in μ . It also proves that the term inside the parentheses is increasing in λ_c , and thus utility is decreasing in λ_c . QED

D Accessory Proofs

Proof of Proposition A.1

We prove the proposition in a series of lemmas. First, let $\bar{\Pi}_i(s_i, \gamma_i; \sigma_{-i})$ denote i 's expected utility for making proposal (s_i, γ_i) if a tie would be broken in her favor. Clearly this is i 's expected utility from making a proposal at any s_i where $-i$ has no atom, and i can always achieve utility arbitrarily close to $\bar{\Pi}_i(s_i, \gamma_i; \sigma_{-i})$ by making ε -higher score proposals. Now $\bar{\Pi}_i(s_i, \gamma_i; \sigma_{-i}) =$

$$-\alpha(s_i + \lambda_v \gamma_i^2) + F_{-i}(s_i) \cdot (\theta - \lambda_c(ix - \gamma_i)^2) + \int_{s_{-i} > s_i} -\lambda_c(ix - \gamma_{-i})^2 d\sigma_{-i}.$$

Taking the derivative with respect to γ_i yields the first Lemma.

Lemma D.1. *At any score s_i where $F_{-i}(\cdot)$ has no atom, the proposal $\left(s_i, i \left(\frac{F_{-i}(s_i)}{\Lambda\alpha + F_{-i}(s_i)}\right) x\right)$ is the strictly best score- s_i proposal, where $\Lambda = \frac{\lambda_v}{\lambda_c}$ is the voter's relative ideological strength. Defining the functions $\gamma_i(y) = iy$ and $y(P) = \frac{P}{\Lambda\alpha + P}x$, the optimal platform may be written as $\gamma_i(y(F_{-i}(s_i)))$.*

Proof: Straightforward and omitted. QED

Lemma D.1 states that at almost every score, proposer i 's best combination of platform and spending to generate that score involves a platform exactly at $i \cdot \left(\frac{F_{-i}(s_i)}{\Lambda\alpha + F_{-i}(s_i)}\right) x = \gamma_i(y(F_{-i}(s_i)))$.

The second lemma establishes that at least one of the proposers is always *active*, in the sense of making a proposal with strictly positive score (all positive-score proposals are positive-quality, but the reverse is not necessarily true). Intuitively, this holds because the proposers wish to move policy in opposite directions from the DM, and can beat negative-score proposals for “free” by proposing the DM's ideal and spending nothing.

Lemma D.2. *In equilibrium $F_k(0) > 0$ for at most one $k \in \{L, R\}$.*

Proof: Suppose not, so $F_i(0) > 0 \forall i$ in some equilibrium. Let U_i^* denote proposer i 's equilibrium utility, which can be achieved by mixing according to her strategy conditional on making score- $s \leq 0$ proposal. Let $\bar{\gamma}^0$ denote the expected ideological outcome and \bar{w}_i^0 the probability proposer i wins conditional on both sides making score ≤ 0 proposals (these quantities may depend on the DM's tie-breaking rule when both make proposals with the

same score). Now $\bar{w}_i^0 \leq 1$ for both candidates and $\theta \geq 0$ (winning office is weakly beneficial). In addition $x_{-1} < 0 < x_1$ implies $-(x_i - \bar{\gamma}^0) \leq -(x_i - 0)$ for at least one k , and both proposers have mean-variance preferences over ideology. Together these imply that $U_k^* \leq \bar{\Pi}_k(0, 0; \sigma_{-k})$, which in turn is $< \bar{\Pi}_k(0, \gamma_k(y(F_{-k}(0))); \sigma_{-k})$ (by Lemma D.1 and $F_{-k}(0) > 0$), so k has a strictly profitable deviation. QED

The third Lemma establishes that in equilibrium there is 0 probability of a tie at any score. The absence of score ties is an intuitive consequence of exactly opposing ideological interests and the fact that spending is “all pay” – at least one proposer will find it in her interests to spend a bit more to break the tie, and make an ideological proposal that is weakly better than the expected outcome from a tie.

Lemma D.3. *In equilibrium there is 0-probability of a tie at any score s .*

Proof: The absence of ties at scores $s \leq 0$ is immediately implied by Lemma D.2. To rule out ties at scores $s > 0$, suppose not, so each proposer’s strategy generates an atom of size $p_i^s > 0$ at some $s > 0$. Now let $\bar{\gamma}_i^s$ denote i ’s expected ideology conditional on a score- s proposal, $\bar{\gamma}^s$ denote the expected ideological outcome conditional on a tie at score s , and \bar{w}_i^s the probability proposer i wins conditional on a tie at score s . Proposer i achieves her equilibrium utility U_i^* by mixing according to her strategy conditional on a score- s proposal. Now using that proposers have mean-variance preferences, that $\theta \geq 0$, and that $-(x_i - \bar{\gamma}^s)^2 \leq -(x_i - 0)^2$ for at least one k , it is straightforward to show that U_k^* is \leq

$$\begin{aligned} & -\alpha(s + \lambda_v[\bar{\gamma}_k^s]^2) + (F_{-k}(s) - p_{-k}^s) \cdot (\theta - \lambda_c(x_k - \bar{\gamma}_k^s)^2) + p_{-k}^s(\bar{w}_k^s\theta - \lambda_c(x_k - 0)^2) \\ & + \int_{s_{-k} > s_k} -\lambda_c(kx - \gamma_{-k})^2 d\sigma_{-k} \end{aligned} \quad (\text{D.1})$$

We now argue k has a strictly profitable deviation. If k ’s proposal at score s is $(s, 0)$ with probability 1, then eqn D.1 is $\leq \bar{\Pi}_k(s, 0; \sigma_{-k}) < \bar{\Pi}_k(s, \gamma_k(y(F_{-k}(s))); \sigma_{-k})$ (by Lemma D.1 and $F_{-k}(s) \geq p_{-k}^s > 0$). If k sometimes proposes something else, then it is straightforward to show that eqn D.1 is $< \left(1 - \frac{p_{-k}}{F_{-k}(s)}\right) \bar{\Pi}_k(s, \bar{\gamma}_k^s; \sigma_{-k}) + \left(\frac{p_{-k}}{F_{-k}(s)}\right) \bar{\Pi}_k(s, 0; \sigma_{-k})$, which is k ’s utility if she were to instead propose $(s, 0)$ with probability $\frac{p_{-k}}{F_{-k}(s)}$, and the expected ideology $\bar{\gamma}_k^s$ of her strategy at score s with the remaining probability (and always win ties). QED

Lemmas D.1 – D.3 jointly imply the “ideological optimality” portion of the Proposition; each proposer makes proposals of the form $(s_i, \gamma_i(y(F_{-i}(s_i))))$ with probability 1. Thus,

proposer i can compute her expected utility from any proposal (s_i, γ_i) as if her opponent only makes proposals of the form $(s_{-i}, \gamma_{-i}(y(F_i(s_{-i}))))$, and the utility from making *any* proposal (s_i, γ_i) where $-i$ has no atom (or a tie would be broken in i 's favor) is therefore $\bar{\Pi}_i^*(s_i, \gamma_i; F) =$

$$-\alpha(s_i + \lambda_v \gamma_i^2) + F_{-i}(s_i) \cdot (\theta - \lambda_c(ix - \gamma_i)^2) + \int_{s_i}^{\infty} -\lambda_c(ix - \gamma_{-i}(y(F_i(s_{-i}))))^2 dF_{-i}. \quad (\text{D.2})$$

Proposer i 's utility from making the *best* proposal with score s_i is $\bar{\Pi}_i^*(s_i, \gamma_i(y(F_{-i}(s)))) ; F$, which we henceforth denote $\bar{\Pi}_i^*(s_i; F)$.

Fourth, we establish that equilibrium score CDFs must satisfy natural properties arising from the all pay component of the contest, and that these properties yield a pair of differential equations characterizing any equilibrium score CDFs $\{F_i, F_{-i}\}$.

Lemma D.4. *The support of the equilibrium score CDFs over $[0, \infty]$ is common, convex, and includes 0; that is, $\text{supp}\{F_i\} \cap [0, \infty] = \text{supp}\{F_{-i}\} \cap [0, \infty] = [0, \bar{s}]$ where $\bar{s} \in (0, \infty]$. In addition, for all i and $s \in [0, \bar{s}]$, $\{F_i, F_{-i}\}$ satisfy the differential equations*

$$f_{-i}(s) \cdot (\theta + \lambda_c((x + y(F_i(s))))^2 - (x - y(F_{-i}(s))))^2 = \alpha$$

Proof: We first argue $\hat{s} > 0$ in support of $F_i \rightarrow F_{-i}(s) < F_{-i}(\hat{s}) \forall s < \hat{s}$. Suppose not; so $\exists s < \hat{s}$ where $-i$ has no atom and $F_{-i}(s) = F_{-i}(\hat{s})$. Then $\bar{\Pi}_i(\hat{s}, \gamma_i; F) - \bar{\Pi}_i(s, \gamma_i; F) = -\alpha_i(\hat{s} - s) < 0$, implying i 's best score- s proposal is strictly better than her best score- \hat{s} proposal, a contradiction. We now argue this yields the desired properties. First, an $\hat{s} > 0$ in i 's support but not $-i$ implies $\exists \delta > 0$ s.t. $F_{-i}(s - \delta) = F_{-i}(s)$. Next, if the common support were not convex or did not include 0, then there would $\exists \hat{s} > 0$ in the common support s.t. neither proposer has support immediately below, and $F_i(s) < F_i(\hat{s}) \forall i, s < \hat{s}$ would imply both proposers have atoms at \hat{s} , contradicting Lemma D.3. Finally $\bar{s} > 0$ follows immediately from Lemma D.2.

To see the differential equations, observe that $\hat{s} > 0$ in $\text{supp}\{F_i\}$ implies all $s \in [0, \hat{s}]$ are also in $\text{supp}\{F_i\}$, implying $\bar{\Pi}_i(s; F) \geq U_i^* \forall s \in [0, \hat{s}]$ and hence $\lim_{s \rightarrow \hat{s}^-} \{\bar{\Pi}_i(s; F)\} \geq U_i^*$. Equilibrium also requires $\bar{\Pi}_i(s; F) \leq U_i^* \forall s$ (otherwise i would have a strictly profitable deviation). Hence $\bar{\Pi}_i(s; F) = U_i^* \forall s \in [0, \bar{s}]$. This further implies the F 's must absolutely continuous over $(0, \infty)$ (given our initial assumptions), and therefore $\frac{\partial}{\partial s}(\bar{\Pi}_i^*(s; F)) = 0$ for almost all $s \in [0, \bar{s}]$. Finally this straightforwardly yields the differential equations for score

optimality, with the boundary conditions implied by Lemma D.4. QED

Proof of Proposition B.1

We prove $F_i(s) = F_{-i}(s) = F(s) \forall s \geq 0$ (and hence $\forall s$ since then $F_i(0) = F_{-i}(0) = 0$ from Lemma D.2) by contradiction. First we explicitly write out the coupled system of differential equations from score optimality, which states that $\forall s \in [0, \bar{s}]$ and $\forall i$:

$$\begin{aligned} f_{-i}(s) \cdot \left(\theta + 2\lambda_c x \sum_j y(F_j(s)) + \lambda_c ([y(F_i(s))]^2 - [y(F_{-i}(s))]^2) \right) &= \alpha \quad \text{and} \\ f_i(s) \cdot \left(\theta + 2\lambda_c x \sum_j y(F_j(s)) - \lambda_c ([y(F_i(s))]^2 - [y(F_{-i}(s))]^2) \right) &= \alpha \end{aligned}$$

It follows immediately that $F_i(s) > F_{-i}(s)$ at $s \geq 0 \rightarrow f_i(s) > f_{-i}(s)$. Restated more intuitively, if $F_i(s) - F_{-i}(s)$ is strictly positive at some $s \geq 0$ then it is also strictly increasing, which further implies that $F_i(s) - F_{-i}(s) > 0 \rightarrow F_i(\hat{s}) - F_{-i}(\hat{s}) \geq F_i(s) - F_{-i}(s) \forall \hat{s} > s$. Now suppose we have non-symmetric score CDFs; then there $\exists \hat{s} \in [0, \bar{s}]$ s.t. $F_i(\hat{s}) > F_{-i}(\hat{s})$; then by the preceding $\lim_{s \rightarrow \bar{s}} \{F_i(s) - F_{-i}(s)\} \geq F_i(\hat{s}) - F_{-i}(\hat{s}) > 0$, contradicting atomless common support. QED

Proof of Proposition B.2

The explicit derivation of strategies is fully contained in Appendix D. It remains only to show that strategies satisfying the necessary conditions in Proposition A.1 and in which the score CDFs are identical ($F_i(s) = F_{-i}(s) = F(s)$ with $F(0) = 0$) are also sufficient for equilibrium. First, the differential equations and boundary conditions imply that $F(s)$ is atomless, so all proposals (s_i, γ_i) yield utility $\bar{\Pi}_i^*(s_i, \gamma_i; F)$ (the utility when ties are broken in i 's favor), and the strictly best proposal $(s_i, \gamma_i(y(F_{-i}(s))))$ with score s_i yields utility $\bar{\Pi}_i^*(s_i; F)$. Thus it suffices to show that $\bar{\Pi}_i^*(s; F) = \max_{s_i} \{\bar{\Pi}_i^*(s_i; F)\} \forall s \in [0, \bar{s}]$. By construction $\bar{\Pi}_i^*(s; F) = \hat{U}_i^* \forall s \in [0, \bar{s}]$. For $s > \bar{s}$ we have $\bar{\Pi}_i(s; F) - \bar{\Pi}_i(\bar{s}; F) = -(\alpha - 1)(s - \bar{s}) < 0 \rightarrow \bar{\Pi}_i(s; F) < \hat{U}_i^*$. For $s < 0$ we have $\bar{\Pi}_i^*(s; F) = \bar{\Pi}_i^*(0; F) \rightarrow \bar{\Pi}_i^*(s; F) = \hat{U}_i^*$. QED.

Details for Footnote 2

To see why spending q_i may equivalently represent the sum of positive campaigning p_i by i to improve her own image and negative campaigning n_i to worsen her opponent's image, suppose that the candidates may engage in these two distinct strategies. Then the DM's utility for electing candidate i is $\mu(p_i - n_{-i}) - \lambda_v \|y_i\|^2$ and her utility for electing candidate $-i$ is $\mu(p_{-i} - n_i) - \lambda_v \|y_{-i}\|^2$. The DM thus prefers candidate i to candidate $-i$ i.f.f.

$$\mu(p_i + n_i) - \lambda_v \|y_i\|^2 \geq \mu(p_{-i} + n_{-i}) - \lambda_v \|y_{-i}\|^2$$

and for the purposes of influencing the DM's behavior positive and negative campaigning are equivalent. In our model, the distinction between positive and negative campaigning would only affect equilibria if they had different marginal costs. The distinction is also meaningful when evaluating voter welfare if valence is interpreted literally, since the candidates view the two strategies equivalently but one increases voter welfare while the other reduces it.

Details for Footnote 5

We explain how the distribution over the final policy outcome depicted in Figure 2 in the main text is derived. Observe that the distance of the final policy outcome from the DM is described by the CDF $[F_Y(y)]^2$ since the DM always chooses candidate with the most distant platform. Thus the density of the distance of the final policy outcome is $2F_Y(y)f_y(y)$. Finally, for each distance $y \in [0, \frac{x}{\Lambda\alpha+1}]$ with total density $2F_Y(y)f_y(y)$ this distance is equally likely to result from a platform of the left or right candidate; thus platforms $\gamma \in [-\frac{x}{\Lambda\alpha+1}, \frac{x}{\Lambda\alpha+1}]$ have density $F_Y(|\gamma|)f_y(|\gamma|)$.